17) $f(x) = \dfrac{x^2}{x^2+2}$

$f'(x) = \dfrac{(x^2+2)\cdot 2x - x^2\cdot 2x}{(x^2+2)^2}$

$f'(x) = \dfrac{4x}{(x^2+2)^2}$ ← Can never $= 0$ → $(x^2+2)^3$

$-$ | $+$ at 0

inc: $[0, \infty)$

dec: $(-\infty, 0]$

$f''(x) = \dfrac{(x^2+2)^2\cdot 4 - 4x\cdot 2(x^2+2)\cdot 2x}{(x^2+2)^4}$

$f''(x) = \dfrac{\cancel{(x^2+2)}[(x^2+2)\cdot 4 - 16x^2]}{(x^2+2)^{\cancel{4}\,3}}$

$f''(x) = \dfrac{8-12x^2}{(x^2+2)^3}$

$8 - 12x^2 = 0$

$12x^2 = 8$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

$-$ | $+$ | $-$ at $-\sqrt{2/3}$, $\sqrt{2/3}$

c.u: $(-\sqrt{2/3}, \sqrt{2/3})$

c.d: $(-\infty, -\sqrt{2/3})(\sqrt{2/3}, \infty)$

inf: $x = \pm\sqrt{2/3}$

19) $f(x) = \sqrt[3]{x+2} = (x+2)^{1/3}$

$f'(x) = \frac{1}{3}(x+2)^{-2/3}$

$f'(x) = \dfrac{1}{3(x+2)^{2/3}}$

even, so double root

$+ \quad\quad +$

$\longleftarrow\!\!\!-\!\!\!-\!\!\!+\!\!\!-\!\!\!-\!\!\!\longrightarrow$

-2

$f''(x) = \frac{-2}{9}(x+2)^{-5/3}$

$f''(x) = \dfrac{-2}{9(x+2)^{+5/3}}$

odd, no double root

$+ \quad\quad -$

$\longleftarrow\!\!\!-\!\!\!-\!\!\!+\!\!\!-\!\!\!-\!\!\!\longrightarrow$

-2

inc: $(-\infty, \infty)$ cu: $(-\infty, -2)$

dec: never cd: $(-2, \infty)$

inf: $x = -2$

31) $f(x) = \sin x \cos x \quad [0, \pi]$

$f'(x) = \sin x \cdot {}^{-}\sin x + \cos x \cdot \cos x$

$f'(x) = \cos^2 x - \sin^2 x = 0$

$\cos^2 x = \sin^2 x$

$\pm \cos x = \pm \sin x$

$x = \pi/4, \; 3\pi/4$

$+ \quad - \quad +$

$\pi/4 \quad 3\pi/4$

$f''(x) = {}^{-}2\cos x \sin x - 2\cos x \sin x$

$f''(x) \; -4\cos x \sin x = 0$

$- \quad +$

$\pi/2$

inc: $[0, \pi/4] \; [3\pi/4, \pi]$

dec: $[\pi/4, 3\pi/4]$

cu: $(\pi/2, \pi)$

cd: $(0, \pi/2)$

inf: $x = \pi/2$

21) $f(x) = x^{1/3}(x+4)$

$f'(x) = x^{1/3}(1) + (x+4)\cdot \frac{1}{3}x^{-2/3}$

$= \frac{3x^{2/3}}{3x^{2/3}} \cdot x^{1/3} + \frac{(x+4)}{3x^{2/3}}$

$= \frac{3x + x + 4}{3x^{2/3}}$

$f'(x) = \frac{4(x+1)}{3x^{2/3}}$

$x = -1$

$x = 0$

$-$ $+$ $+$

-1 0

inc: $[-1, \infty)$

dec: $(-\infty, -1]$

$f''(x) = \frac{3x^{2/3}\cdot 4 - (4x+4)\cdot 2x^{-1/3}}{9x^{4/3}}$

$= \frac{\frac{x^{1/3}}{x^{1/3}}\left(12x^{2/3} - \frac{(8x+8)}{x^{1/3}}\right)}{9x^{4/3}}$

$= \frac{12x - 8x - 8}{9x^{5/3}}$

$= \frac{4(x-2)}{9x^{5/3}}$

$+$ $-$ $+$

0 2

cu: $(-\infty, 0)(2, \infty)$

cd: $(0, 2)$

inf: $x = 0, 2$

23) $f(x) = e^{-x^2/2}$

$f'(x) = e^{-x^2/2} \cdot -x$

$f'(x) = \dfrac{-x}{e^{x^2/2}}$

$x = 0$

$+ \quad -$

0

$f''(x) = -e^{-x^2/2} + -x \cdot e^{-x^2/2} \cdot -x$

$f''(x) = \dfrac{-1 + x^2}{e^{x^2/2}}$

$x = 1, -1$

$+ \quad - \quad +$

$-1 \quad 1$

inc: $(-\infty, 0)$ cu: $(-\infty, -1)(1, \infty)$

dec: $(0, \infty)$ cd: $(-1, 1)$

inf: $x = 1, -1$